# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

## MTMADSE06T-MATHEMATICS (DSE3/4)

### MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No. 1 and any *five* from the rest**

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Define Virial of coplanar forces acting on a body.

(b) If two forces act on a particle and the particle undergoes a small displacement, show that the total work done by those forces is equal to the work done by their resultant.

(c) When frictional force arises between two static rough bodies, in contact with each other, then what is the relation between the angle of friction and the co-efficient of friction?

(d) If $V$ is the potential energy of a body in equilibrium under the action of some forces, state the conditions indicating the equilibrium is stable or unstable.

(e) If the moment of inertia of a circular disc of mass $M$ and radius $a$ about any of its diameter is $\frac{Ma^2}{4}$, find the moment of inertia about a line passing through the centre and perpendicular to the plane of the disc.

(f) Define kinetically equivalent systems. State the necessary and sufficient condition that two systems be kinetically equivalent.

(g) If a rigid body as compound pendulum swings under gravity about a fixed horizontal axis, then write the expression of the length of equivalent pendulum. Define all the terms correctly.

(h) Find the velocity of an artificial satellite of the earth, given $g = 9.8\ \text{m/s}^2$ and radius of the earth $= 6.4\times10^6$ m.

2. (a) Two uniform rods, $AB$ and $CD$ each of weight '$W$' and length '$a$' are smoothly jointed at $O$ where $OB = OD = b$. The rods rest in a vertical plane with the ends $A$ and $C$ on a smooth table and the ends $B$ and $D$ are connected by a light string. Show that the reaction at the joint is $\frac{aW\tan\alpha}{2b}$, where $\alpha$ is the inclination of either rod to the vertical. 4

(b) Find the centre of gravity of the arc of the cardioid $r = a(1+\cos\theta)$ lying above the initial line. 4

3. (a) Two equal uniform rods of length $l$ jointed at one end so that the angle between them is $\theta$ and they rest in a vertical plane on a smooth sphere of radius $R$. Show that the rods are in unstable or stable equilibrium according as 4

$$l \lesseqgtr 4R\,\mathrm{cosec}\,\theta$$

(b) A semi-circular disc rests in a vertical plane with its curved edge on a rough horizontal plane and rough vertical plane, where $\mu$ and $\mu'$ are the coefficients of frictions at the horizontal plane and vertical plane respectively. Show that the greatest angle that the bounding diameter makes with the vertical plane is 4

$$\cos^{-1}\left(\frac{3\mu\pi}{4}\cdot\frac{1+\mu'}{1+\mu\mu'}\right).$$

4. (a) Find the equation of Poinsot's central axis for any system of forces in three dimensions. 4

(b) A force $F$ acts along the axis of $z$, and a force $mF$ along a straight line, intersecting the axis of $x$ at a distance $c$ from the origin and parallel to the plane of $yz$. Show that as this straight line turns around the axis of $x$, the central axis of the forces generate the surface $\{m^2z^2+(m^2-1)y^2\}(c-x)^2=x^2z^2$. 4

5. (a) Deduce the differential equation of a central orbit under a central force in pedal form. 4

(b) If a particle describes a nearly circular path of radius $1/c$ under the influence of a central force $\mu\varphi(u)$ (where, $u=\frac{1}{r}$, $r$ being the distance of the particle at any instant from the centre of force), find the condition that this may be a stable motion. 4

6. (a) If a planet were suddenly stopped in its orbit, supposed circular, then show that it would fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution. 4

(b) The motion of a point relative to a fixed frame is defined by $x=a\cos\omega t$, $y=b\sin\omega t$. Show that the motion of the point represented in a moving frame with the same origin will describe a circle, if the frame revolves in a positive sense with angular velocity $\omega$. 4

7. Find whether a straight line is at any point of its length, a principal axis of a given material system. If so, find the direction of the other two principal axes. 8

Hence show that through each point of a plane lamina there exists a pair of principal axes of the lamina.

8. (a) Show that the kinetic energy of a rigid body of mass $M$ moving in two dimensions is equal to the sum of kinetic energy of a particle of mass $M$ placed at the centre of inertia and moving with it and the kinetic energy of the body relative to the centre of inertia. 4

(b) A uniform rod of length $2a$ is placed with one end in contact with a smooth horizontal table and is then allowed to fall; if $\alpha$ be its initial inclination to the vertical, show that its angular velocity is 4

$$\sqrt{\frac{6g}{a}\cdot\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}},$$

when it is inclined at angle $\theta$.

9. (a) A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding. Show that the centre of the sphere moves with a constant acceleration $\frac{5}{7}g\sin\alpha$ down the plane and for pure rolling $\mu>\frac{2}{7}\tan\alpha$, 5

where $\alpha$ is the inclination of the plane to the horizontal, $\mu$ is the coefficient of friction and $g$ is the acceleration due to gravity.

(b) A heavy circular disc is revolving in a horizontal plane about its centre, which is fixed. An insect of mass $\frac{1}{n}$ th that of the disc walks from the centre along a radius and then flies away. Show that the final angular velocity is $\frac{n}{n+2}$ times the original angular velocity of the disc. 3

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